

Understanding quarkonium polarization

- General considerations on the study of di-fermion decays of $J = 1$ states
 - The role of the choice of the reference frame
 - The interplay between production and decay kinematics
 - A frame-invariant formalism for polarization measurements
- “Messages” for polarization analyses (and calculations)

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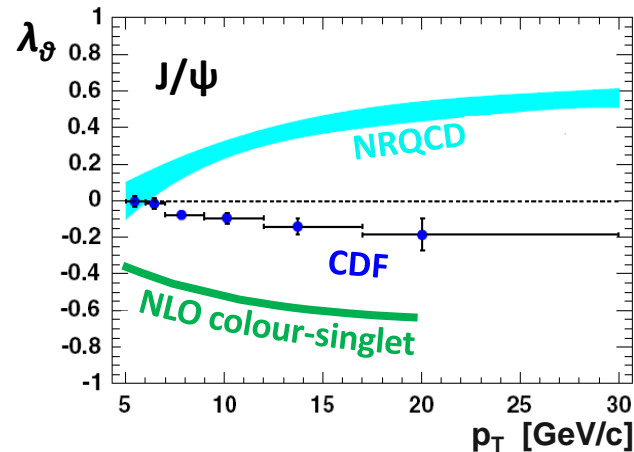
in collaboration with Carlos Lourenço, João Seixas, Hermine Wöhri

QWG Meeting, Fermilab, May 19th, 2010

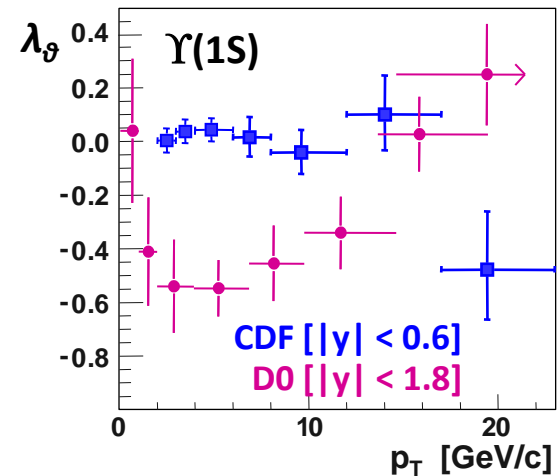
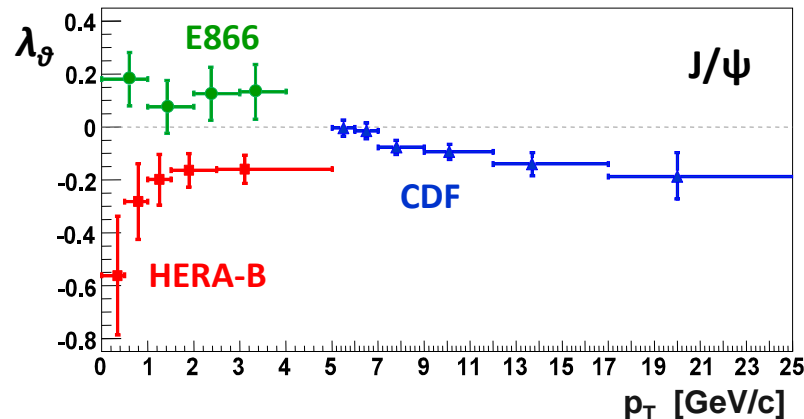


Motivation

- No well-confirmed model yet



- Measurements are challenging, the present status is puzzling



We believe that we can achieve some progress... going back to the fundamentals!

Basics

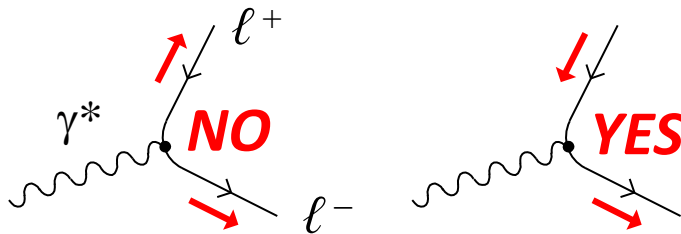
$J = 1 \rightarrow$ three J_z eigenstates $|1, +1\rangle$, $|1, 0\rangle$, $|1, -1\rangle$ wrt a certain z

Measure polarization = measure (average) angular momentum composition

Method: study the (dilepton) **decay angular distribution** in the $Q\bar{Q}$ rest frame

Relation between angular momentum state and decay distribution comes from:

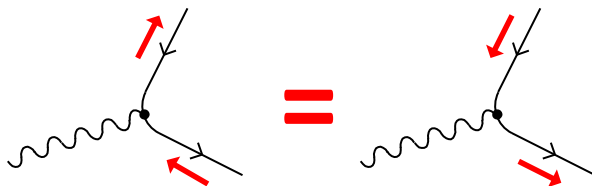
1) “helicity conservation”



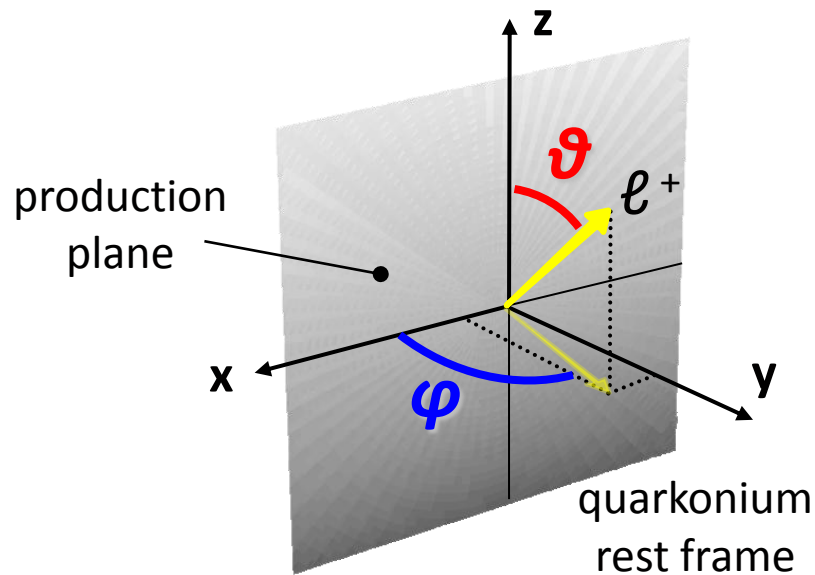
2) rotational covariance of angular momentum eigenstates

$$\frac{1}{2} |1, +1\rangle + \frac{1}{2} |1, -1\rangle - \frac{1}{\sqrt{2}} |1, 0\rangle$$

3) parity conservation



The general distribution



$$\frac{dN}{d\Omega} \propto 1 + \lambda_{\theta} \cos^2 \theta + \lambda_{\phi} \sin^2 \theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos \phi$$

$$+ \lambda_{\phi}^{\perp} \sin^2 \theta \sin 2\phi + \lambda_{\theta\phi}^{\perp} \sin 2\theta \sin \phi$$

asymmetric by reflection about the production plane
 \rightarrow must be zero in *inclusive* measurements

“Unpolarized” J/ψ does not exist

Single elementary subprocess: $|\psi\rangle = a_{-1} |1, -1\rangle + a_0 |1, 0\rangle + a_{+1} |1, +1\rangle$

$$\frac{dN}{d\Omega} \propto 1 + \lambda_\theta \cos^2\theta + \lambda_\phi \sin^2\theta \cos 2\phi + \lambda_{\theta\phi} \sin 2\theta \cos \phi + \dots$$

$$\frac{1 - 3|a_0|^2}{1 + |a_0|^2}$$

$$\frac{2 \operatorname{Re} a_{+1}^* a_{-1}}{1 + |a_0|^2}$$

$$\frac{\sqrt{2} \operatorname{Re}[a_0^*(a_{+1} - a_{-1})]}{1 + |a_0|^2}$$

There is no combination of a_0 , a_{+1} and a_{-1} such that $\lambda_\theta = \lambda_\phi = \lambda_{\theta\phi} = 0$

The angular distribution is never intrinsically isotropic

Only a fortunate **mixture of subprocesses** (or randomization effects) can lead to a cancellation of all three *measured* anisotropy parameters

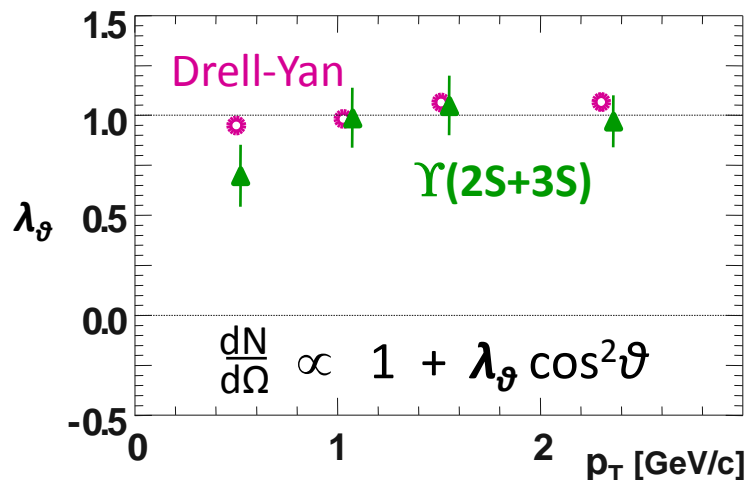
→ Polarization is a “necessary” property of $J = 1$ states
Measuring and understanding it is crucial

... also from an “experimental” point of view:
quarkonium acceptances depend strongly on the dilepton decay kinematics. Quarkonium is by default unpolarized in MC generators...



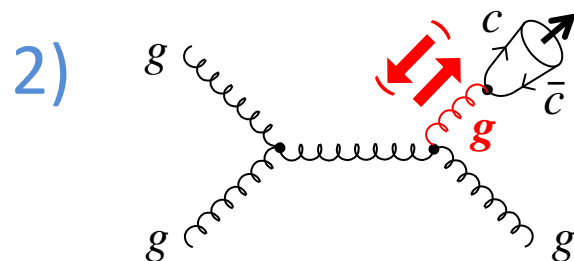
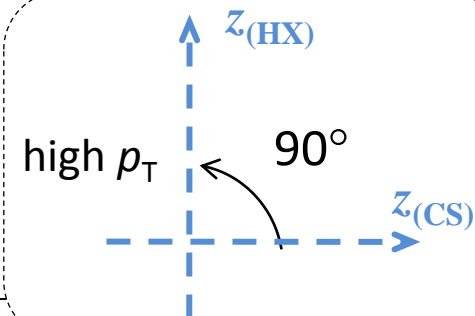
What polarization axis?

- 1) helicity conservation (at the *production* vertex)
 $\rightarrow J=1$ states produced in *fermion-antifermion annihilations* ($q\bar{q}$ or e^+e^-)
 at Born level have transverse polarization along the
relative direction of the colliding fermions (Collins-Soper axis)



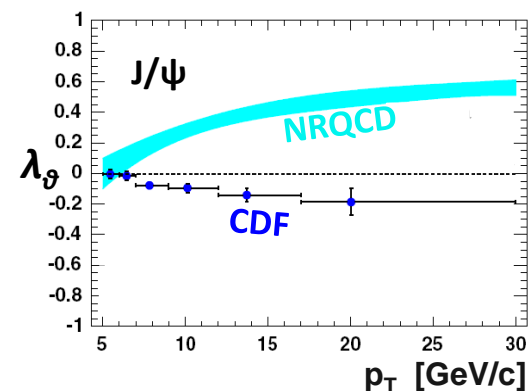
Drell-Yan is a paradigmatic case
 but not the only one

E866
 Collins-Soper frame



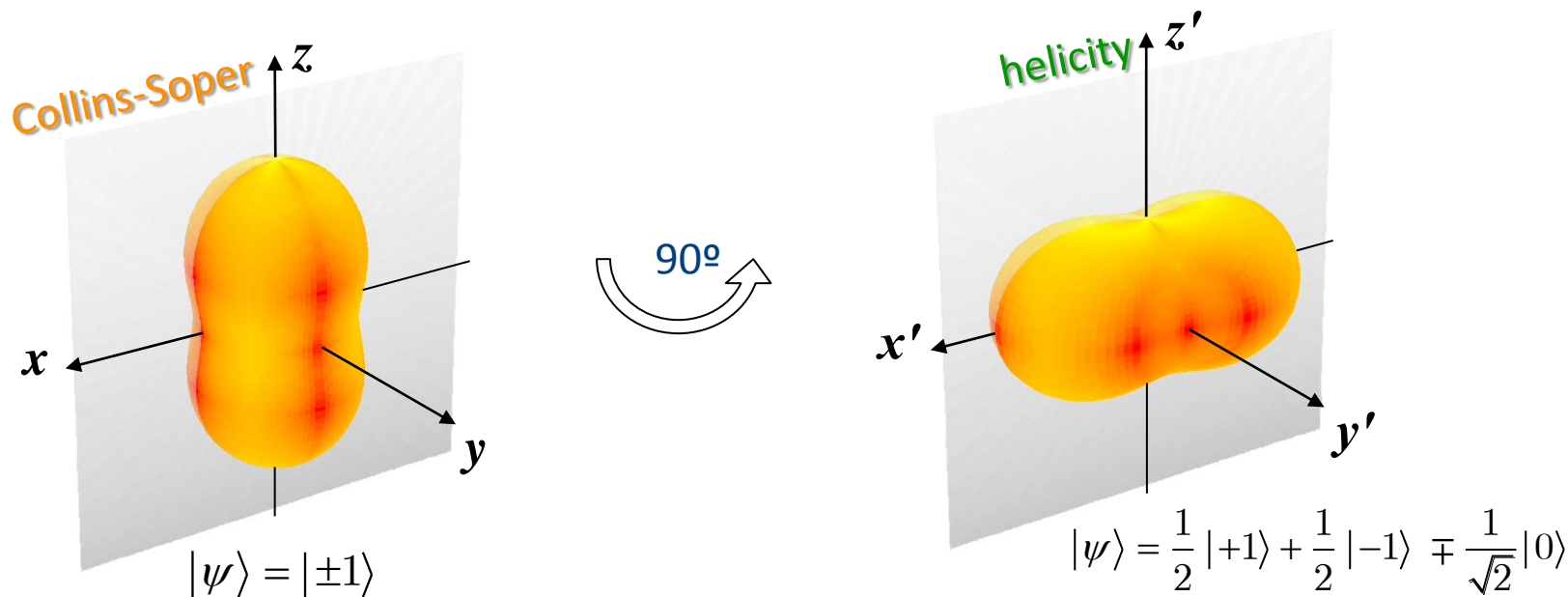
NRQCD \rightarrow at *very large* p_T ,
 quarkonium produced from the
 fragmentation of an
 on-shell gluon, inheriting
 its natural spin alignment

\rightarrow large, transverse polarization
 along the $Q\bar{Q}$ (=gluon) **momentum (helicity axis)**



The observed polarization depends on the frame

For $|p_L| \ll p_T$, the CS and HX frames differ by a rotation of 90°

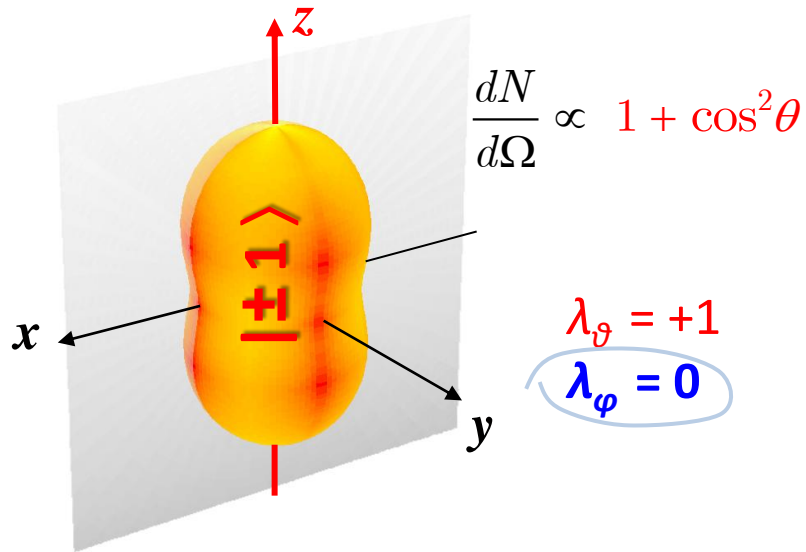


$$\frac{dN}{d\Omega} \propto 1 + \cos^2 \theta$$

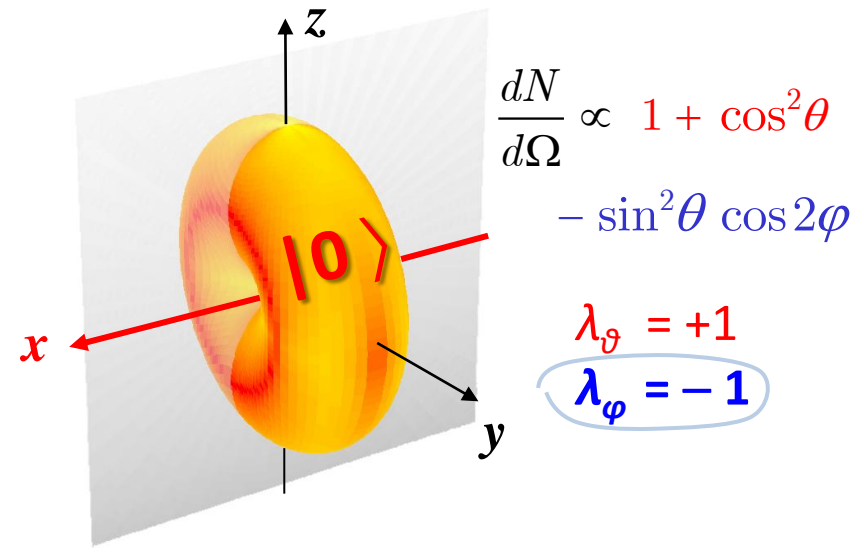
$$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3} \cos^2 \theta + \frac{1}{3} \sin^2 \theta \cos 2\varphi$$

The azimuthal anisotropy is not a detail

Case 1: natural **transverse** polarization



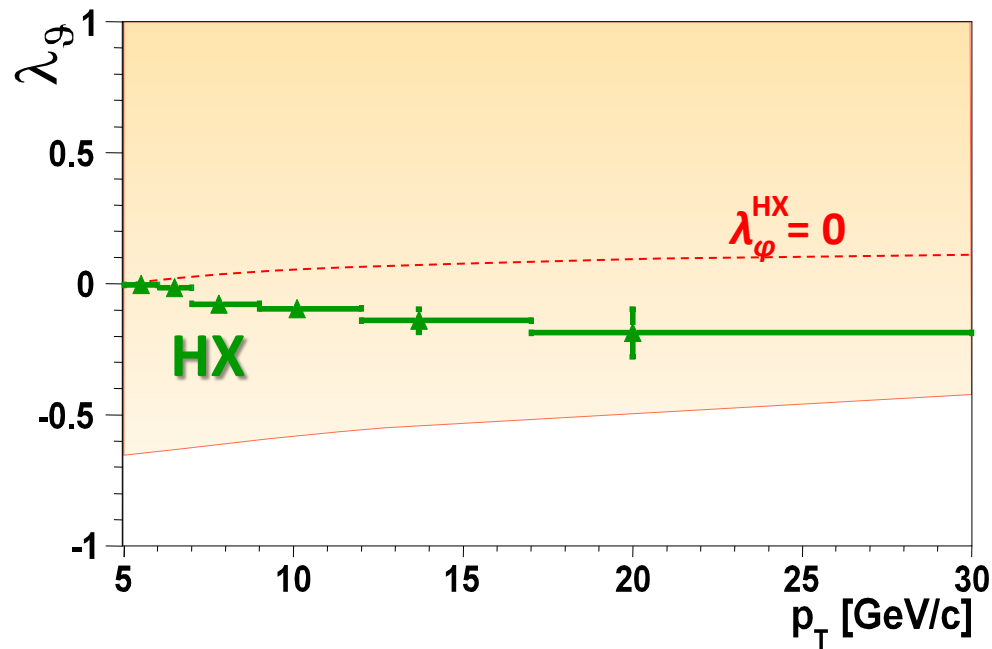
Case 2: natural **longitudinal** polarization, observation frame \perp to the natural one



These two decay distributions are indistinguishable when the azimuthal dependence is integrated out. But they correspond to opposite *natural* polarizations, which can only be originated by completely different production mechanisms.

In general, measurements not reporting the azimuthal anisotropy provide an incomplete physical result. Their fundamental interpretation is impossible (relies on arbitrary assumptions).

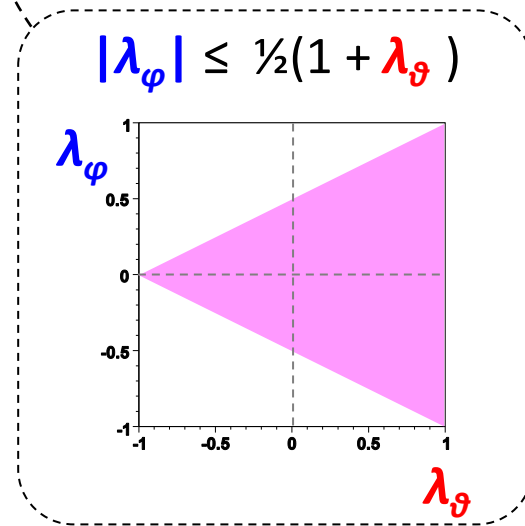
How would the CDF J/ψ result look like in the CS frame?



Without information on the azimuthal anisotropy, we cannot translate λ_g from one frame to another

CS

for all possible hypotheses on λ_ϕ^{HX}



One hypothesis

HERA-B

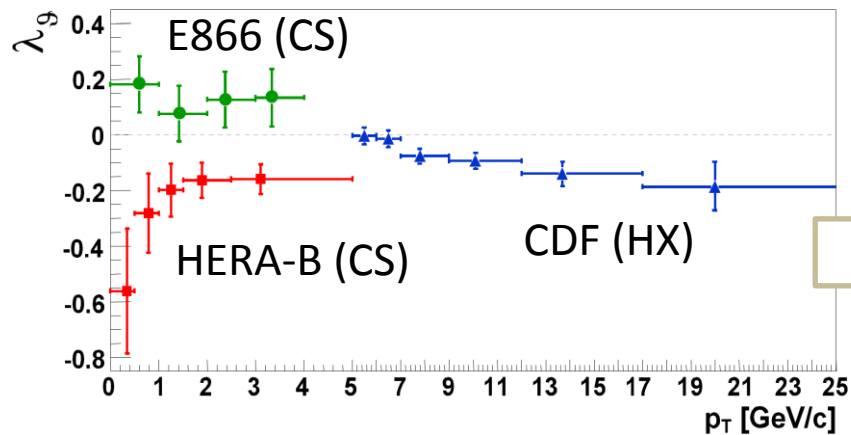
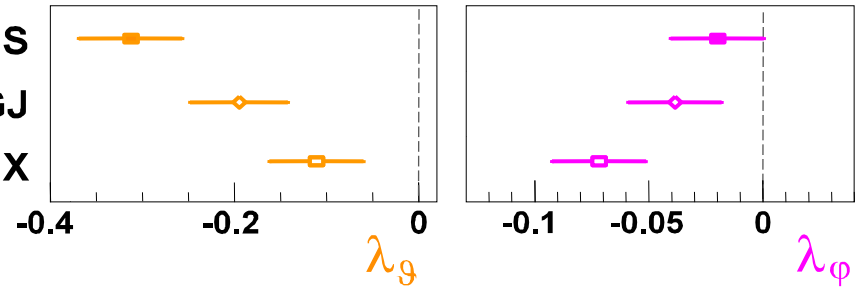
J/ψ 's naturally polarized in the CS frame

- most significant λ_θ
- purely polar anisotropy, $\lambda_\phi \sim 0$

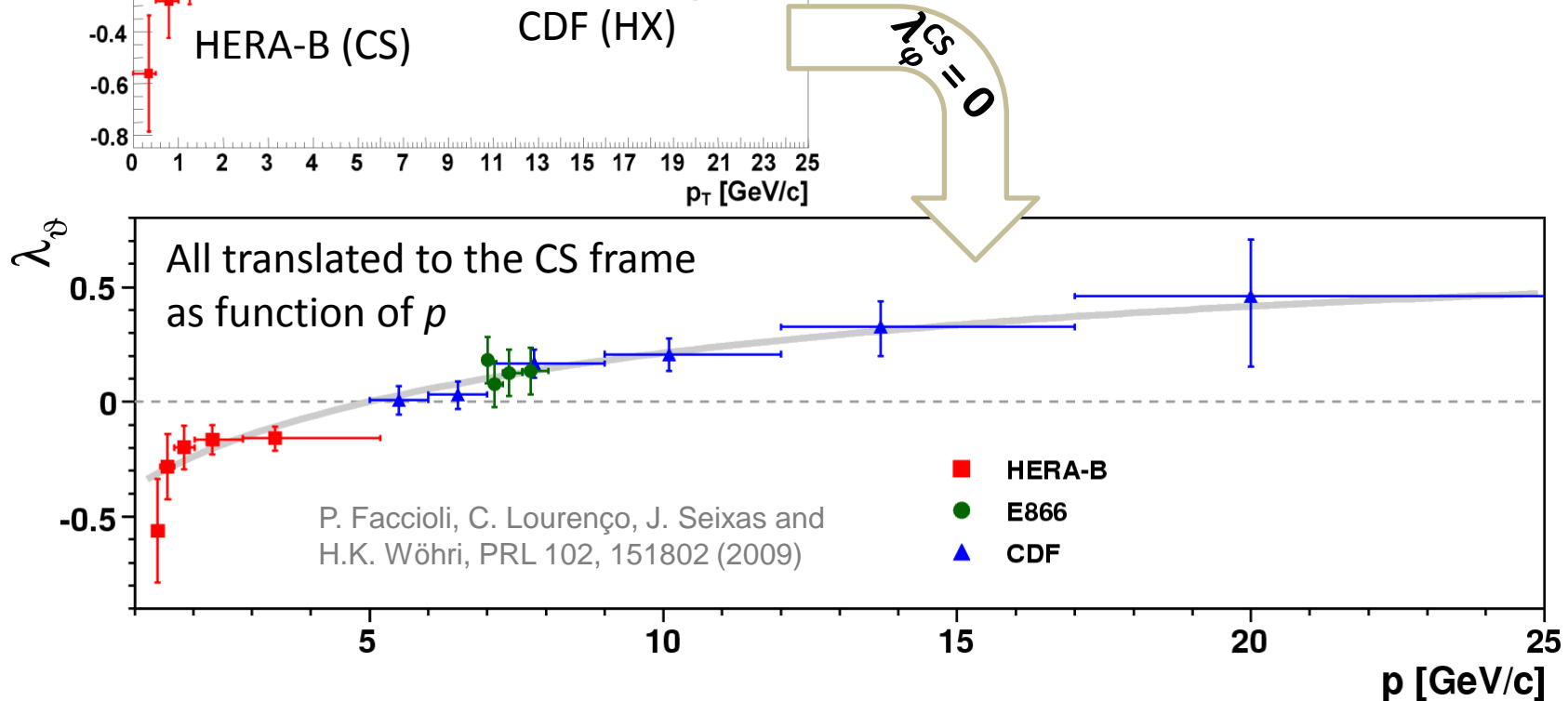
CS

GJ

HX



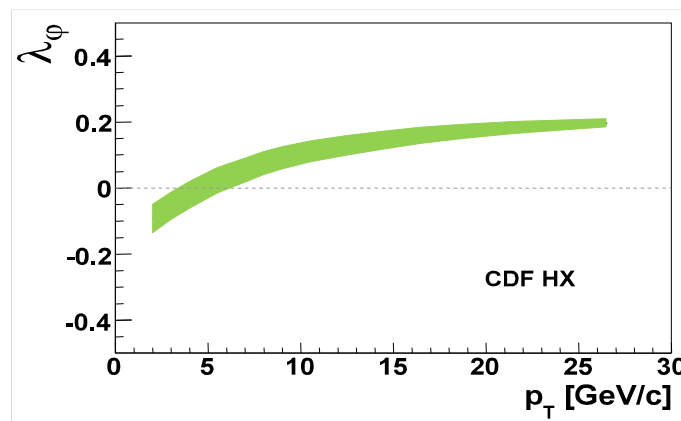
Assuming that this continues to be valid up to collider energies



Message n°1

Today, we are allowed to make the speculation in the previous slide because CDF has not reported the azimuthal anisotropy.

We have assumed that $\lambda_\varphi = 0$ in the CS frame. This automatically implies that a significant value of λ_φ should be measured in the HX frame:

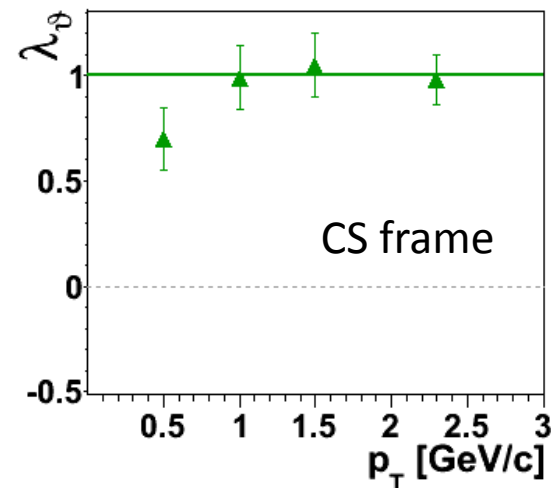


By measuring also λ_φ CDF will remove this ambiguity of interpretation.

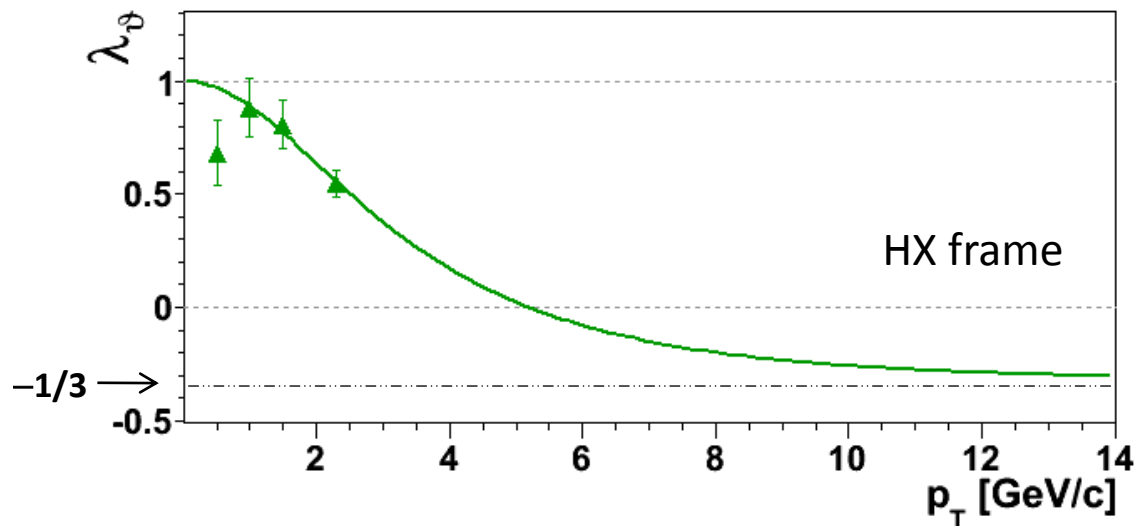
Measure the *full* angular decay distribution, not only the polar anisotropy.

What if E866 had chosen the helicity frame?

E866's $\Upsilon(2S+3S)$ result is compatible with the Drell-Yan "template" $1 + \cos^2\theta_{CS}$



In the helicity frame λ_θ would be seen as strongly kinematics-dependent:



Reference frames are not all equally good

How the anisotropy parameters transform from one frame to another depends *explicitly* on the production kinematics. In fact, the angle δ between helicity and Collins-Soper axes is given by

$$\cos\delta = \frac{m p_{\perp}}{m_{\top} p}$$

Gedankenscenario:

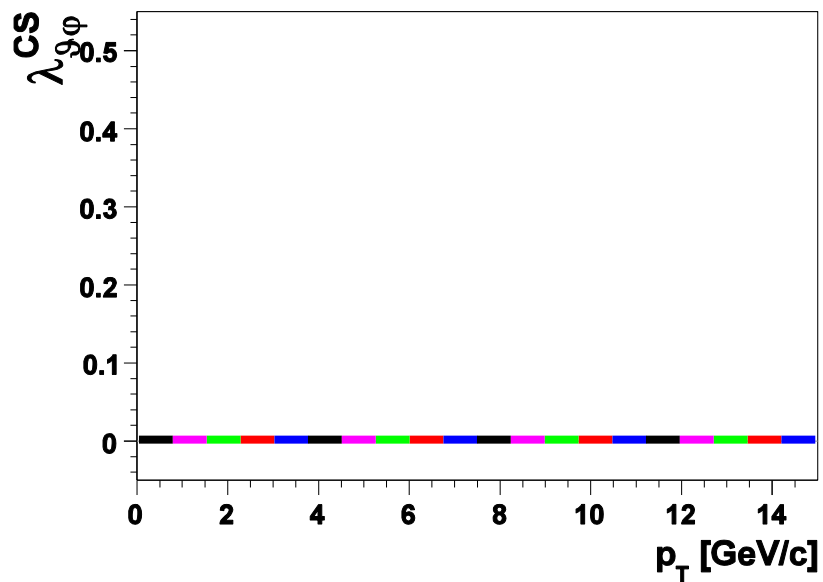
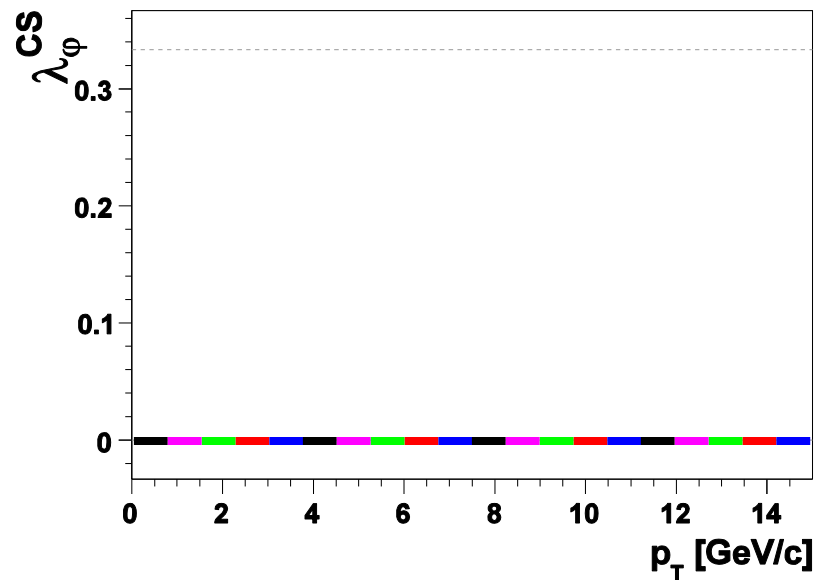
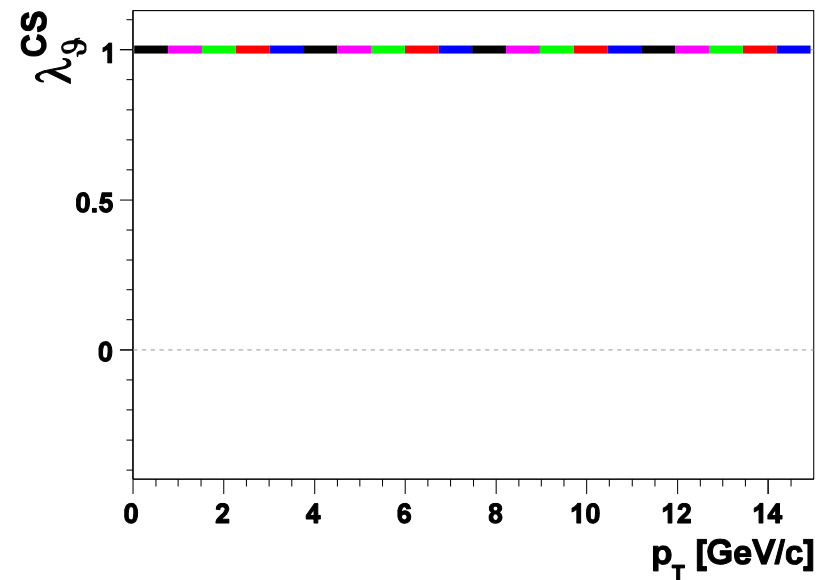
how would different experiments observe a Drell-Yan-like decay distribution
 [“naturally” of the kind $1 + \cos^2\vartheta$ in the Collins-Soper frame]
with an arbitrary choice of the reference frame?

We consider Υ decay. For simplicity of illustration we assume that each experiment has a flat acceptance in its nominal rapidity range:

CDF	$ y < 0.6$
D0	$ y < 1.8$
ATLAS & CMS	$ y < 2.5$
ALICE e^+e^-	$ y < 0.9$
ALICE $\mu^+\mu^-$	$2.5 < y < 4$
LHCb	$2 < y < 5$

The lucky frame choice

(CS in this case)



ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

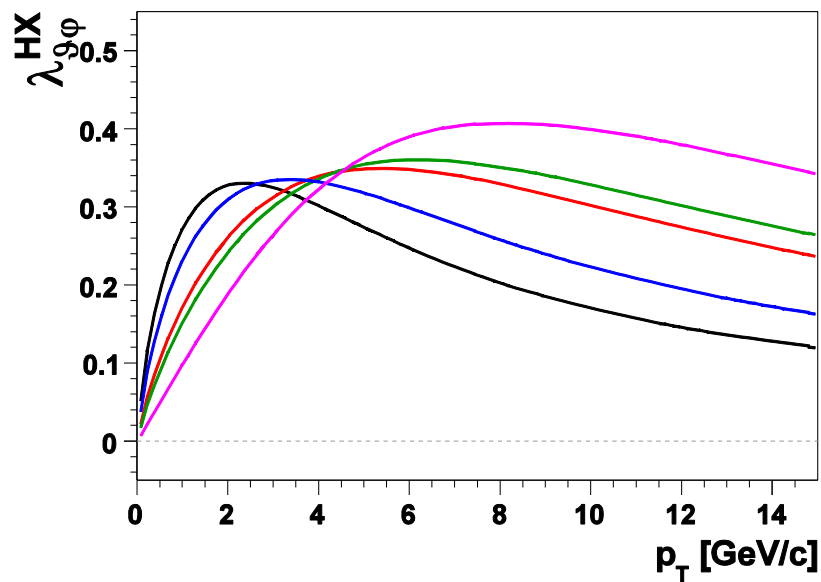
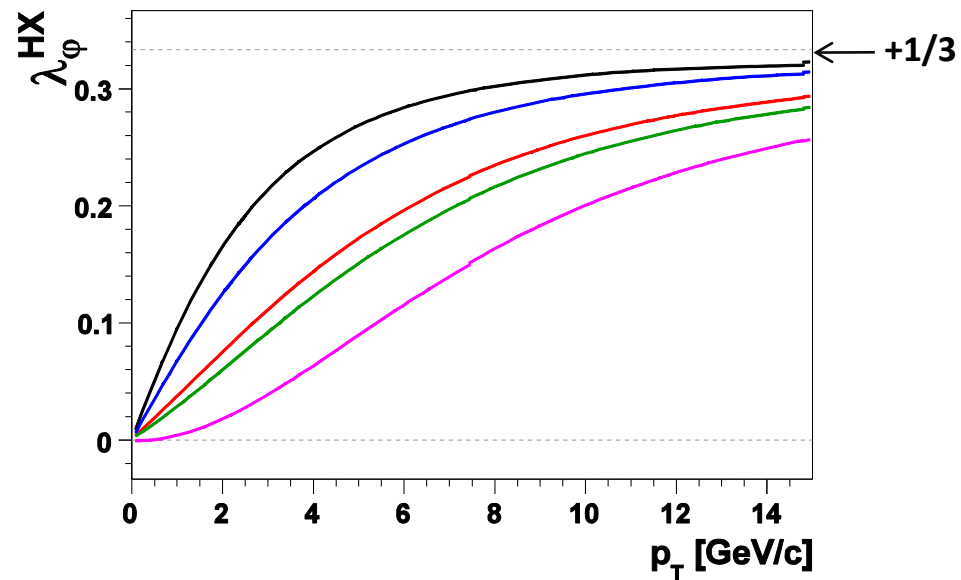
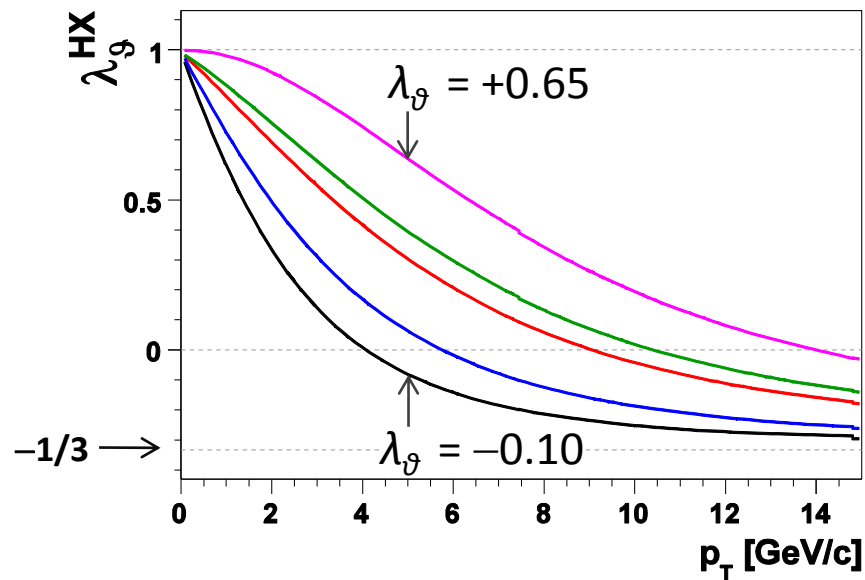
D0

ALICE e^+e^-

CDF

Less lucky choice

(HX in this case)



ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

D0

ALICE e^+e^-

CDF

Message nº2

When observed in an arbitrarily chosen frame, the simplest possible pattern of a constant natural polarization may be seen as a complex decay distribution rapidly changing with p_T and rapidity. This is not wrong, but gives a misleading view of the phenomenon, even inducing an artificial dependence of the measurement on the specific kinematic window of the experiment.

Measure in more than one frame.

Message nº3

Warning: *transformed* (= not natural) polarization depends not only on the acceptance *interval*, but also on the acceptance *shape*!

The problem can be solved by measuring in *small* kinematic cells.

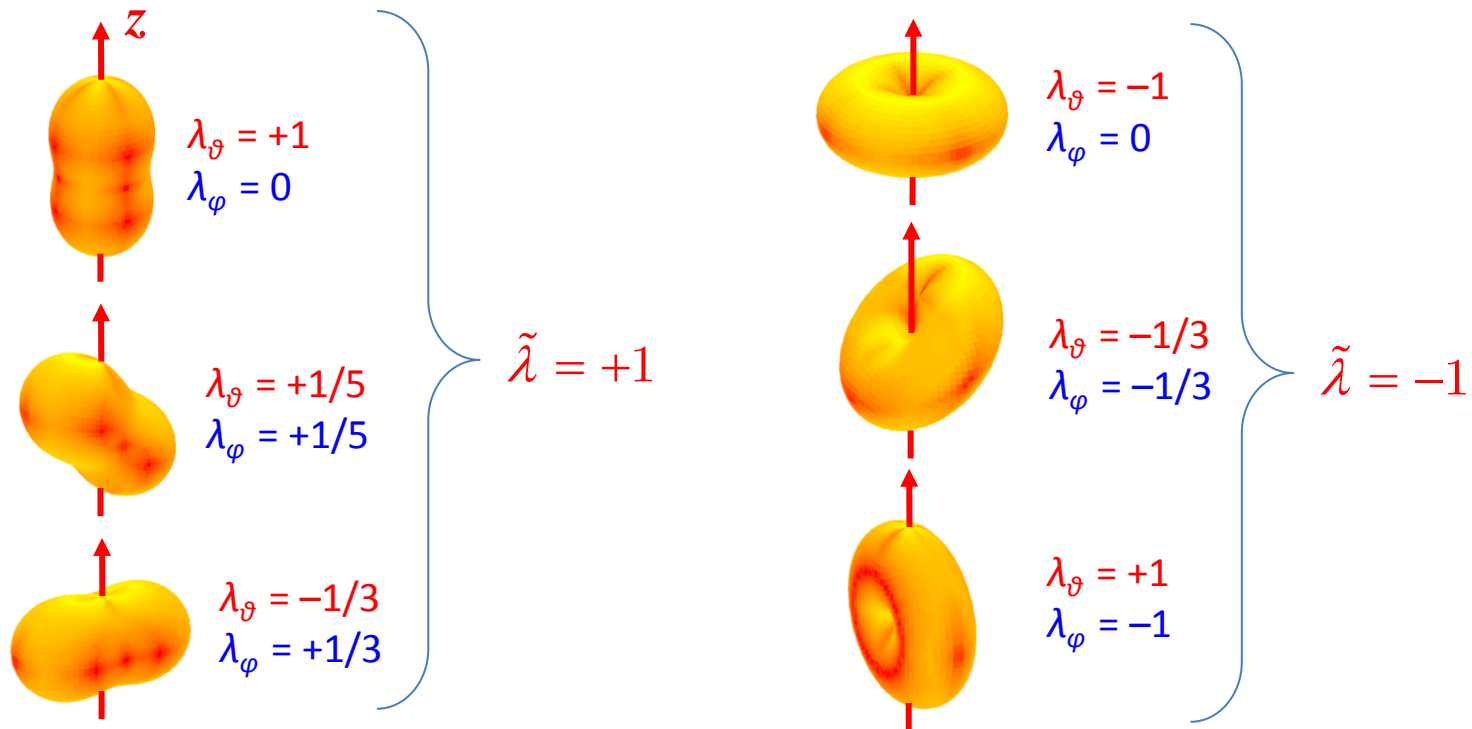
Also *theoretical calculations* should take into account how the momentum distribution is distorted by the acceptance of the specific experiment, or provide *event-level* predictions.

Avoid (as much as possible) kinematic averages.

Frame-independent polarization

The *shape* of the distribution is obviously frame-invariant.

→ there exists a family of frame-independent quantities, e.g. $\tilde{\lambda} = \frac{\lambda_{\vartheta} + 3\lambda_{\varphi}}{1 - \lambda_{\varphi}}$ (and any function of it)



Measuring frame-invariant quantities is useful for

- a self-consistency check of the analysis (is $\tilde{\lambda}$ really the same in two frames?)
- a clearer representation of the results, removing frame-induced kinematic dependencies

Basic meaning of the frame-invariant quantities

Let us suppose that, in the collected events, n different elementary subprocesses yield angular momentum states of the kind

$$|\psi^{(i)}\rangle = a_{-1}^{(i)} |1, -1\rangle + a_0^{(i)} |1, 0\rangle + a_{+1}^{(i)} |1, +1\rangle, \quad i = 1, 2, \dots, n$$

(wrt a given quantization axis), each one with probability $f^{(i)}$ ($\sum f^{(i)} = 1$).

The **rotational properties of J=1 angular momentum states** $[d_{+1,M}^1(\theta) + d_{-1,M}^1(\theta) = \delta_{|M|,1}]$ imply that

the combinations $a_{+1}^{(i)} + a_{-1}^{(i)}$ are independent of the choice of the quantization axis

The quantity
$$\mathcal{F} = \sum_{i=1}^n f^{(i)} \mathcal{F}^{(i)} = \frac{1}{2} \sum_{i=1}^n f^{(i)} |a_{+1}^{(i)} + a_{-1}^{(i)}|^2 \quad (0 \leq \mathcal{F} \leq 1)$$

is therefore frame-independent. It can be shown to be equal to

$$\mathcal{F} = \frac{1 + \lambda_g + 2\lambda_\phi}{3 + \lambda_g} \quad (\text{note: } \mathcal{F} = \frac{1 + \tilde{\lambda}}{3 + \tilde{\lambda}})$$

In other words, there always exists a calculable frame-invariant relation of the form

$$(1 - \mathcal{F})\lambda_g + 2\lambda_\phi = 3\mathcal{F} - 1$$

The Lam-Tung limit

Another consequence of rotational properties of angular momentum eigenstates:

for each state $|\psi^{(i)}\rangle = a_0^{(i)} |0\rangle + a_{+1}^{(i)} | +1\rangle + a_{-1}^{(i)} | -1\rangle$

there exists a quantization axis z' wrt which $a_0^{(i)'} = 0$

→ dilepton produced in each single elementary subprocess has a distribution of the type

$$\lambda_g^{(i)'} = +1, \quad \lambda_\varphi^{(i)'} = 2F^{(i)} - 1, \quad \lambda_{g\varphi}^{(i)'} = 0 \quad (F^{(i)} = \frac{1}{2} |a_{+1}^{(i)} + a_{-1}^{(i)}|^2)$$

wrt its specific “ $a_0^{(i)'} = 0$ ” axis.

Case $F^{(i)} = \frac{1}{2}$: each subprocess is characterized by a *fully transverse* polarization

$$\lambda_g^{(i)'} = +1, \quad \lambda_\varphi^{(i)'} = 0, \quad \lambda_{g\varphi}^{(i)'} = 0$$

wrt a certain “*natural*” axis (which may be different from subprocess to subprocess).

$$\rightarrow F = \sum f^{(i)} F^{(i)} = \frac{1}{2} = \frac{1 + \lambda_g + 2\lambda_\varphi}{3 + \lambda_g}$$

$$\rightarrow \lambda_g + 4\lambda_\varphi = 1 \quad \text{Lam-Tung identity} \\ \text{(Drell-Yan including pQCD corrections)}$$

Simple interpretation of the LT relation

1. The *existence (and frame-independence)* of the LT relation is the *kinematic* consequence of the rotational properties of $J = 1$ angular momentum eigenstates
2. Its *form* derives from the *dynamical* input that all contributing processes produce the dilepton via one *transversely* polarized photon

More generally:

- Corrections to the Lam-Tung relation (parton- k_T , higher-twist effects) should continue to yield *invariant* relations.

In the literature, deviations are often searched in the form

$$\lambda_g + 4\lambda_\phi = 1 - \Delta$$

But this is not a frame-independent relation. Rather, corrections should be searched in the invariant form

$$\mathcal{F} = 1/2 (1 - \Delta_{\text{inv}}) \quad \rightarrow \quad \lambda_g(1 + \Delta_{\text{inv}}) + 4\lambda_\phi = 1 - 3\Delta_{\text{inv}}$$

- For *any* superposition of processes, concerning *any* $J = 1$ particle (even in parity-violating cases: W, Z), we can always calculate a *frame-invariant* relation analogous to the LT relation.

Advantages

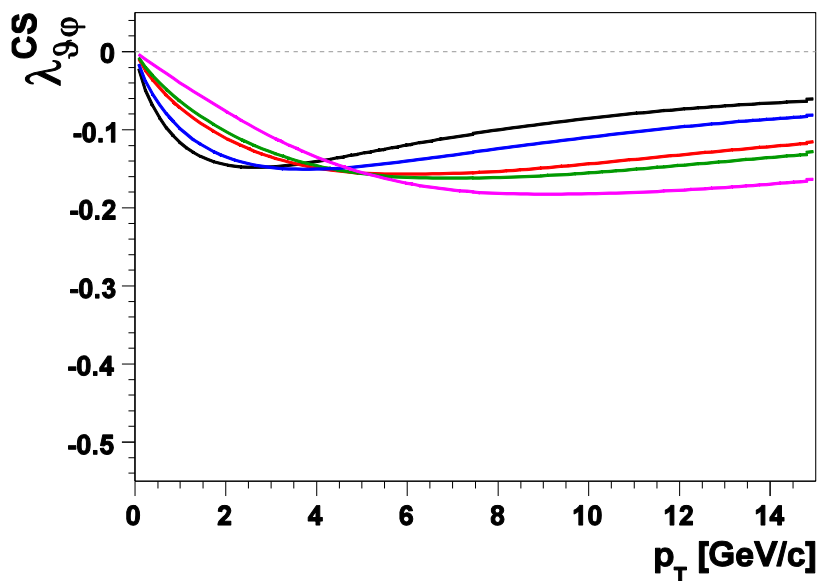
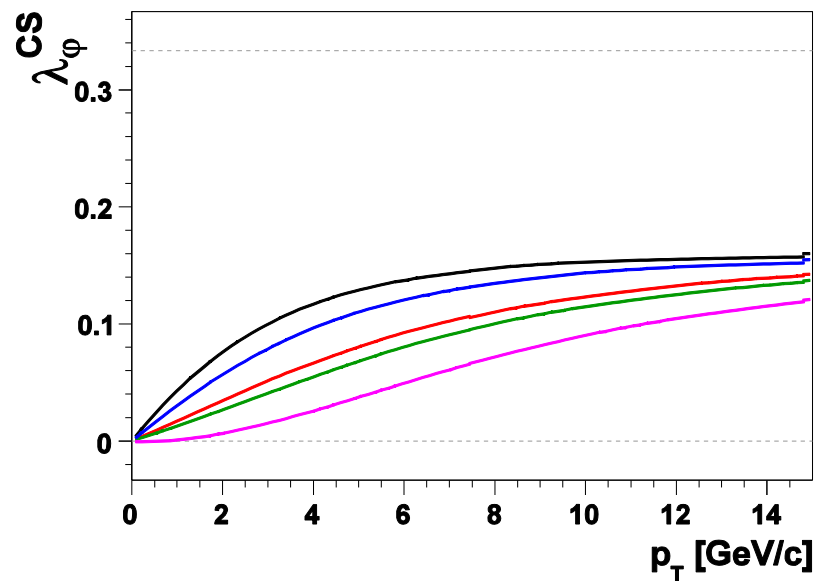
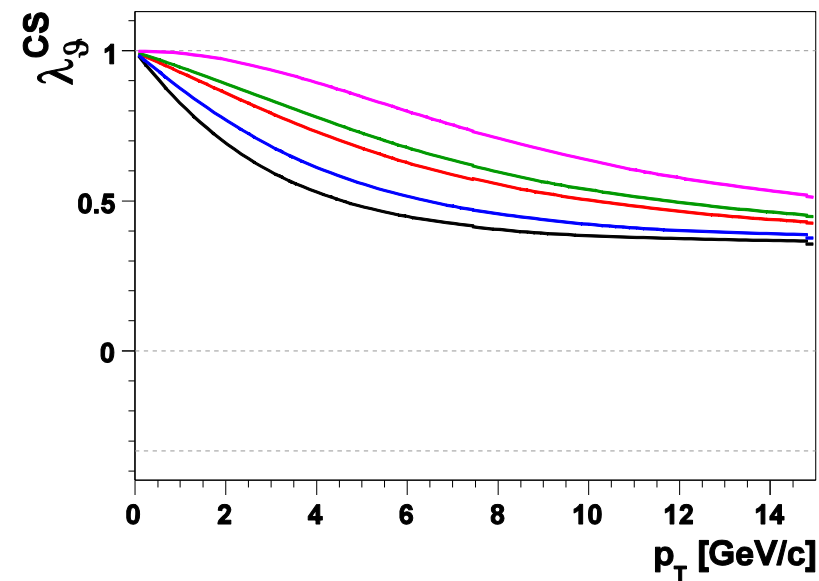
Invariant quantities provide an easier representation of polarization results.

Let us consider, for illustrative purposes, the following (purely hypothetical) mixture of subprocesses for Υ production:

- 1) $f^{(1)} = 60\%$ of the events have a natural **transverse** polarization in the **CS** frame
- 2) $f^{(2)} = 40\%$ of the events have a natural **transverse** polarization in the **HX** frame

Frame choice 1

All experiments choose the CS frame



ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

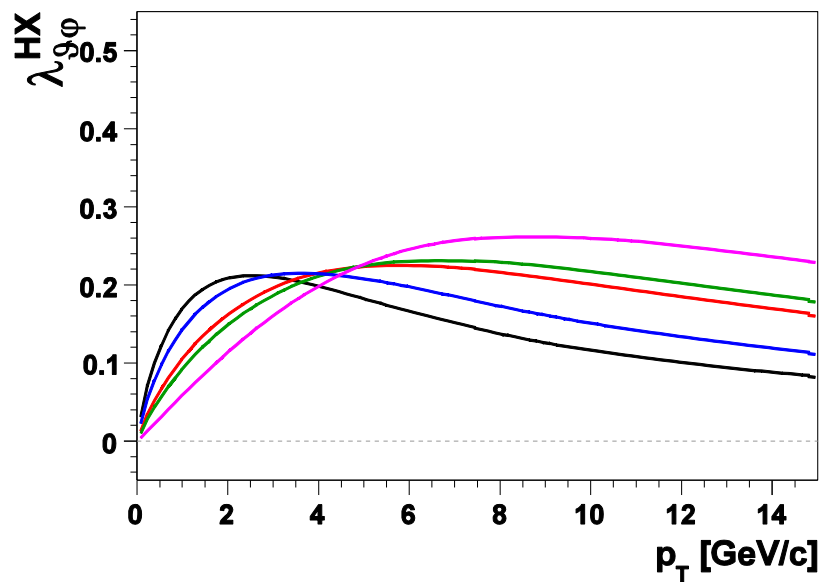
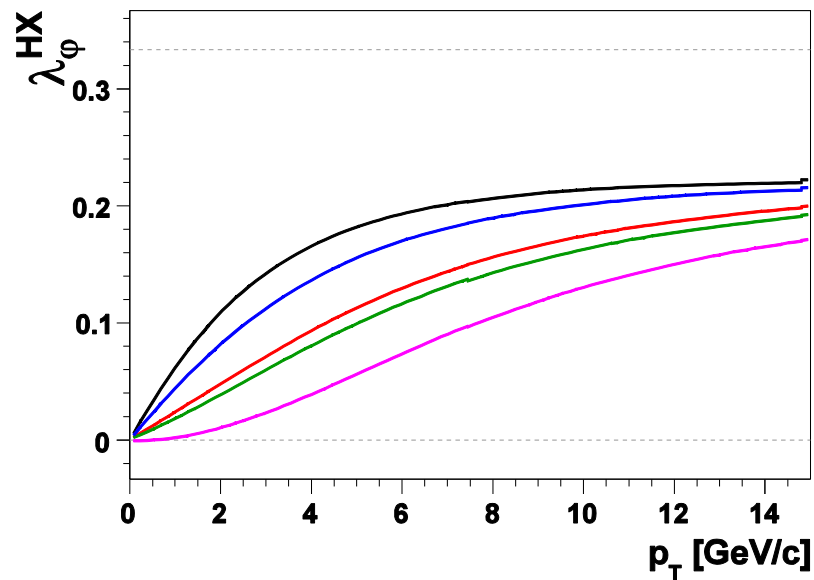
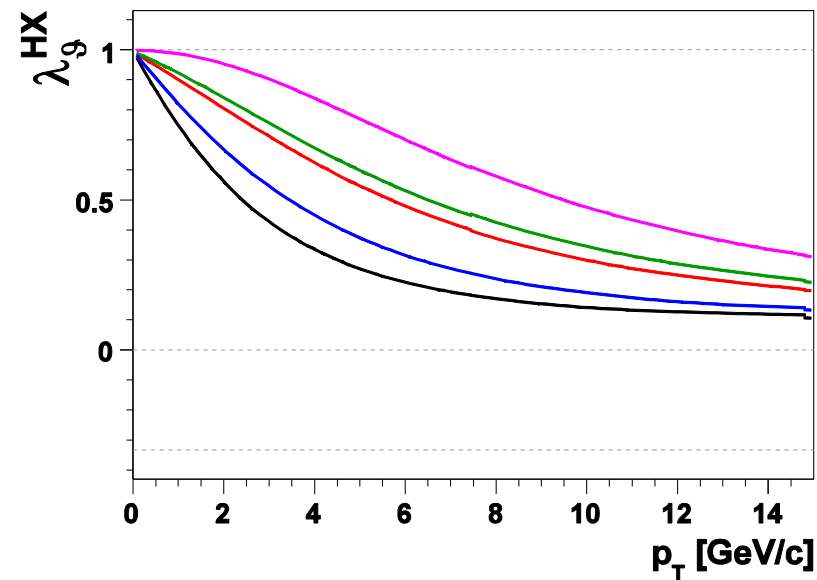
D0

ALICE e^+e^-

CDF

Frame choice 2

All experiments choose the HX frame



ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

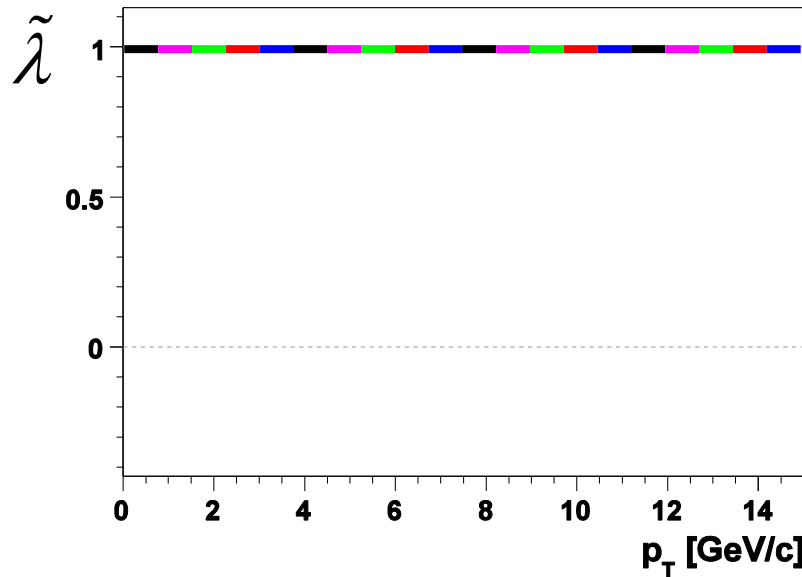
D0

ALICE e^+e^-

CDF

Any frame choice

The experiments measure an invariant quantity, for example $\tilde{\lambda} = \frac{\lambda_g + 3\lambda_\varphi}{1 - \lambda_\varphi}$



ALICE $\mu^+\mu^-$ / LHCb

ATLAS / CMS

D0

ALICE e^+e^-

CDF

$\tilde{\lambda}$ is an “average of the natural polarizations”,
irrespective of the directions of the respective axes:

$f^{(i)}$ = statistical weight of the i -th process

$\lambda_g^{*(i)}$ = i -th “natural” polarization

$$\tilde{\lambda} = \frac{\sum_{i=1}^n \frac{f^{(i)}}{3 + \lambda_g^{*(i)}} \lambda_g^{*(i)}}{\sum_{i=1}^n \frac{f^{(i)}}{3 + \lambda_g^{*(i)}}}$$

Message nº4

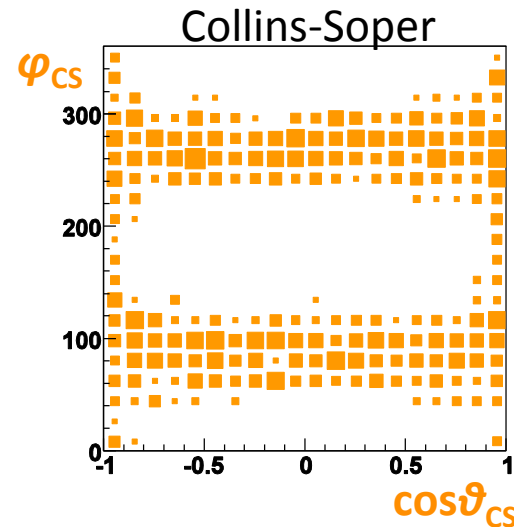
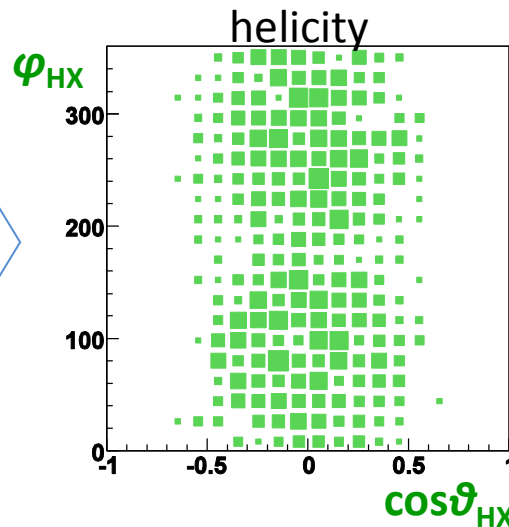
Frame-invariant quantities are immune to “extrinsic” kinematic dependencies induced by the observation perspective.

They minimize the acceptance-dependence of the measurement.

Use invariant relations to facilitate comparisons.

Experimental biases are not frame-invariant

Minimum
detector
sensitivity to
muon momenta
+ trigger cuts



Reconstructed
unpolarized $\Upsilon(1S)$

CMS-like MC with
 $p_T(\mu) > 3 \text{ GeV}/c$
(both muons)

$p_T(\Upsilon) > 10 \text{ GeV}/c$,
 $|\eta(\Upsilon)| < 1$,

This spurious “polarization” must be accurately corrected.

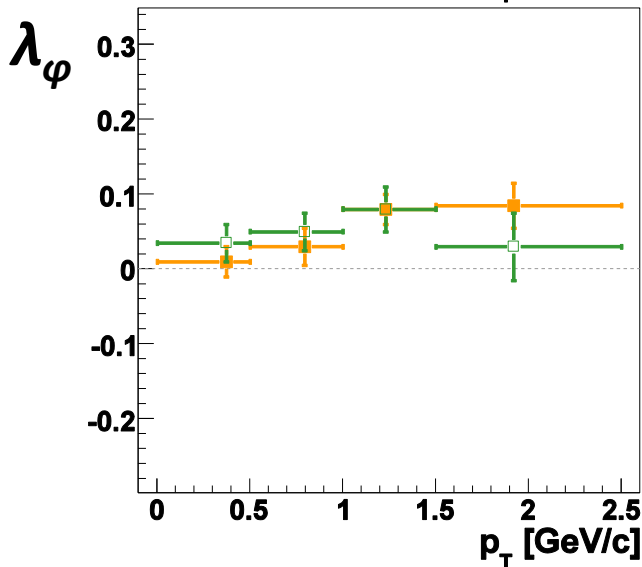
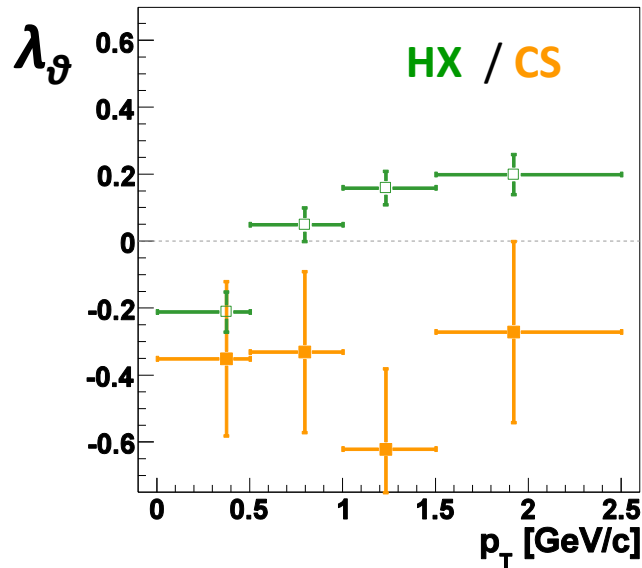
The “detector polarization frame” is naturally defined in the LAB

The induced anisotropies have not the properties of a $J = 1$ decay distribution

- unaccounted detector effects due to acceptance limitations will
violate the physical frame-invariant relations between decay angular parameters.
- checking whether the same value of an invariant quantity is obtained
(within systematic errors) *in two distinct polarization frames* is a non-trivial test.

Example

Example of preliminary J/ψ result, before evaluation of systematic errors



Is this a self-consistent pattern?

→ check quantitatively by calculating the average invariant “polarization”

$$\tilde{\lambda} = \frac{\lambda_g + 3\lambda_\phi}{1 - \lambda_\phi}$$

$$\tilde{\lambda}(\text{HX}) - \tilde{\lambda}(\text{CS}) = 0.49 [\pm 0.13]$$

[CS and HX data fully statistically correlated]

order of magnitude of the expected systematic error on the anisotropy parameters

Message nº5

**Use invariant relations
for a better control over systematic effects.**

Summary

- Even if experimentally challenging, polarization measurements are textbook exercises of basic quantum mechanics. By keeping in mind fundamental notions we will perform better polarization measurements
- The observable angular distribution reflects the rotational-covariance properties of angular momentum
 - it depends (strongly) on the reference frame according to definite rules
 - its parameters satisfy a frame-independent identity, a special case of which is the Lam-Tung relation
- In the quarkonium analyses of CMS, we will
 - determine the *full* angular decay distribution, not only the polar anisotropy
 - provide results in two polarization frames
 - avoid averages over large kinematic intervals, using (p_T, y) cells
 - exploit the existence of frame-independent relations
 - to detect residual systematic effects
 - to facilitate the comparison with theoretical calculations and other results